Analytic Combinatorics for Multiple Object Tracking

Intended Audience.
The intended audience is any engineer, Ph.D. student, and interested person working in multi-object tracking and data fusion. The development should be of special interest to individuals working in what is often called random finite sets (or finite point processes), and those working on large problems requiring principled approximations. Open discussion of problems and specific interests are welcome.

Rationale
Exact solutions of many problems in tracking have high computational complexity and are impractical for all but the smallest of problems. Practical implementations entail approximation. There is a bewildering variety of established trackers available and practicing engineers and/or researchers often study them almost in isolation of each other without fully understanding what these trackers are about and how they are inter-related. One reason for this is that these filters have different combinatorial problems which are approached by explicitly enumerating the feasible solutions. The enumeration is usually a highly detailed, hard to understand accounting scheme specific to the filter and the details cloud understanding the filter and make it hard to compare different filters. On the other hand, the analytic combinatoric approach presented in this tutorial avoids the heavy accounting burden and provides a solid tool to work with. This tool is the derivative of multivariate calculus, which all engineers easily understand.

This tutorial is designed to facilitate understanding of the classical theory of Analytic Combinatorics (AC) and how to apply it to problems in multi-object tracking. AC is an economical technique for encoding combinatorial problems—without information loss—into the derivatives of a generating function (GF). Exact Bayesian filters derived from the GF avoid the heavy accounting burden required by traditional enumeration methods. Although AC is an established mathematical field, it is not widely known in either the academic engineering community or the practicing data fusion/tracking community. This tutorial lays the groundwork for understanding the methods of AC, starting with the GF for the classical Bayes-Markov filter. From this cornerstone, we derive many established filters (e.g., PDA, JPDA, JIPDA, PHD, CPHD, MultiBernoulli, MHT) with simplicity, economy, and insight. We also show how to use the saddle point method (method of stationary phase) to find low complexity approximations of probability distributions and summary statistics.

Previous Tutorials:
This tutorial was presented at below listed conferences with an average of 10 attendees. Given the recent popularity of the subject, approximately 10 registration is expected at 2023 Fusion Conference.

- 20th Int. Conference on Information Fusion, Xi'an, China, July 10–13, 2017.
- 21st Int. Conference on Information Fusion, Cambridge, UK, July 10–13, 2018.
- 22nd Int. Conference on Information Fusion, Ottawa, Canada, July 2–5, 2019.
- 2020 IEEE Radar Conference, Florence, Italy, September 21-25, 2020 (virtual event, video presentation).

- 2021 IEEE Radar Conference, Atlanta, GA, USA, May 10-14, 2021 (virtual event, video presentation).
- 24th Int. Conference on Information Fusion, Sun City, South Africa, November 1–4, 2021 (virtual event, video presentation).
- 2022 IEEE Radar Conference, New York City, NY, USA, March 21-25, 2022.
- 25th Int. Conference on Information Fusion, Linköping, Sweden, July 4–7, 2022.

Prerequisites

First course in probability and/or signal processing.

Length of Tutorial

Half a day (three and a half hours)

Tentative Outline

- Part 1 — ***Why Analytic Combinatorics (AC)***? (Approximately 45min)
  – Generating functions and functionals

- Part 2 — ***Lower Level Fusion – Target Tracking*** (Approximately 45min)
  – Tracking
  – Pointillist family: JPDA, JIPDA, PHD/CPHD & more

- Part 3 — ***Examples*** (Approximately 45min)
  – Joint Intensity Filter (JiFi) for extended objects
  – Joint Intensity Filter (JiFi) for multiple objects
  – Palm and object-to-object correlation

- Part 4 — ***Approximations*** (Approximately 45min)
  – Blazing saddle points and derivatives

- Part 5 — ***Concluding remarks*** (Approximately 15min)

Learning Outcomes

Understanding of analytic combinatorics and its use in target tracking
Understanding of radar target tracking
Understanding of classical and modern target tracking algorithms
Connecting theory to practice

Presenter(s)

Roy L. Streit, R. Blair Angle and Murat Efe

Biographies

Roy Streit Senior Scientist, Metron, Reston, Virginia, and Professor (Adjunct) of Electrical and Computer Engineering, University of Massachusetts–Dartmouth. IEEE Fellow. IEEE AESS Board of Governors, 2016-18. President, ISIF, 2012. Research interests include multi-target tracking, multi-sensor data fusion, medical imaging, signal processing, pharmacovigilance, and business analytics. Author, *Poisson Point Processes*, Springer, 2010 (Chinese translation, Science Press, 2013). Co-author, *Bayesian Multiple Target Tracking*, 2nd Edition, Artech, 2014. Seven US patents. He is the co-author of the book entitled *Analytic*

Combinatorics for Multiple Object Tracking, Springer, scheduled to be published in December 2020.

Blair Angle is a senior research scientist at Metron, Inc. Since joining Metron in 2008, he has worked as the technical lead on a variety of projects involving mathematical and statistical modeling, machine learning, tracking, simulation, signal processing, and software development. During his tenure at Metron, he has written or co-written several proposals for DARPA, ONR, etc. which have led to new Metron funding and research. His current research involves multiple-object tracking, with a focus on applying analytic combinatorial (AC) methods to data association problems. Along with Dr. Roy Streit, he recently developed and implemented a working version of the Multisensor JiFi (JPDA intensity Filter), a multisensor, multiobject tracking filter for extended objects. He is the co-author of the book entitled *Analytic Combinatorics for Multiple Object Tracking*, Springer, scheduled to be published in December 2020.

Murat Efe Senior IEEE, Professor of Electrical and Electronics Engineering at Ankara University. Numerous papers in refereed journals, conferences, and seminars on target tracking/data fusion. He was an Associate Editor for IEEE Transactions on Aerospace and Electronic Systems between 2008 and 2021 and was one of the lecturers for the NATO-CSO Lecture Series entitled "Radar and SAR Systems for Airborne and Space-based Surveillance and Reconnaissance" between 2013-2017 where a total of 13 countries, namely Italy, UK, France, Spain, Germany, Romania, US, Canada, Portugal, Lithuania, Bulgaria, Poland and Australia were visited for these lectures. Dr. Efe is a technical consultant to a number of defense companies on tracking and fusion related projects. Also he served on the executive board of the Electrical, Electronics and Informatics Research Group of the Scientific and Technological Research Council of Turkey. Dr. Efe is a member of Board of Directors of ISIF for the term 2014-2016 and again for the term 2017-2019 and 2020-2023. He is the co-author of the book entitled *Analytic Combinatorics for Multiple Object Tracking*, Springer, scheduled to be published in December 2020.

Most Recent Publications

1. Streit, R. L., Angle, R. B. And Efe, M., "Analytic Combinatorics for Multiple Object Tracking", Springer, 2020 (http://www.springer.com/9783030611903).

2. Angle, R. B., Streit, R. L. and Efe, M., "Multiple Target Tracking With Unresolved Measurements", IEEE Signal Processing Letters, Vol 28, pp. 319 - 323. (2021)

3. Angle, R. B., Streit, R. L. and Efe, M., "A Low Computational Complexity JPDA Filter with Superposition", IEEE Signal Processing Letters, Vol **28**, pp. 1031-1035. (2021)